Classifying homomorphism-homogeneous algebras

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(joint work with Igor Dolinka and Éva Jungábel)

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In 2006 P. J. Cameron and J. Nešetřil introduced a relaxed version of homogeneity which does not generalize, but is closely related to, the classical notion of ultrahomogeneity: a structure is homomorphism-homogeneous if every homomorphism between finitely generated substructures of the structure extends to an endomorphism of the structure.

Although considerable attention has been payed to the classification and general properties of homomorphism-homogeneous relational structures, not much is known about homomorphism-homogeneous algebras. Under a different name, finite homomorphism-homogeneous groups have been fully classified already in 1979, and that was the only attempt in this direction in past 30 years.

In this talk we consider homomorphism-homogeneous monounary algebras, lattices and homomorphism-homogeneous semilattices (understood as algebras). We present the characterization of homomorphism-homogeneous monounary algebras and lattices, and then provide several classes of homomorphism-homogeneous semilattices and several classes of semilattices which are not homomorphism-homogeneous. Note that the characterization of homomorphism-homogeneous semilattices is still an open problem.

Finally, we show that in the class of all finite algebras of a given finite algebraic type which contains at least one binary operation, homomorphism-homogeneous objects cannot be classified in a satisfactory manner.

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